Amplitude-independent chaotic synchronization and communication

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ABSTRACT

One problem with using chaotic synchronization to communicate is that the response system is nonlinear, so that any variation in the amplitude of the chaotic driving signal degrades synchronization of the response system to the drive system. In this work it is shown that it is possible to design a response system that reproduces a scaled version of the chaotic driving signal when the drive signal is attenuated or amplified. A simple communications system is demonstrated to show that this type of synchronization is useful, and the effects of noise on the communications system are studied.

Keywords: chaos, communication, spread spectrum, synchronization

1. INTRODUCTION

Synchronizing of chaotic systems that are coupled by a one-way driving is a popular research topic, with much speculation on possible applications to communications [1-14] One problem in using a chaotic signal as an information carrier is the problem of amplitude distortion, or fading. If a chaotic signal is broadcast, for example, there will be some attenuation between transmitter and receiver. Since the response system which is driven by the transmitted chaotic signal is nonlinear, changing the amplitude of the chaotic driving signal will throw the response system out of synchronization. It is possible to use a linear response system, but this eliminates the possibility of cascading response systems [3, 4], which makes transmitting and detecting information much easier.

This work shows how to build a simple chaotic system so that the response system is not sensitive to the amplitude of the driving signal, but is still nonlinear. It is also shown that this type of system can still be used to send and receive information. More detailed results will be reported in a longer paper[15].

2. THEORY OF SYNCHRONIZATION

The theory of the synchronization of chaotic systems is described in detail elsewhere [2], so only a brief description is included here. We begin with a dynamical system that may be described by the ordinary differential equation

$$\dot{u}(t) = f(u) \tag{1}$$

The system is then divided into two subsystems, $u = (v,w)$;

$$\begin{aligned} \dot{v} &= g(v,w) \\ \dot{w} &= h(v,w) \end{aligned} \tag{2}$$

where $v=(u_1,...,u_m)$, $g=(f_1(u),...,f_m(u))$, $w=(u_{m+1},...,u_n)$, and $h=(f_{m+1}(u),...,f_n(u))$. The division is truly arbitrary since the reordering of the u_i variables before assigning them to *v*, *w*, *g*, and h is allowed.

A first response system may be created by duplicating a new sub-system *w′* identical to the *w* system, substitute the set of variables *v* for the corresponding *v′* in the function *h*, and augment Eqs. (2) with this new system, giving,

$$\begin{aligned} \dot{v} &= g(v,w) \\ \dot{w} &= h(v,w)\ . \\ \dot{w}' &= h(v,w') \end{aligned} \tag{3}$$

If all the Lyapunov exponents of the *w′* system (as it is driven) are less than zero, then $w' - w \rightarrow 0$ as $t \rightarrow \infty$.

It is possible to take this system further. One may also reproduce the *v* subsystem and drive it with the *w′* variable [3, 4], giving

$$\begin{aligned} \dot{v} &= g(v,w) \\ \dot{w} &= h(v,w) \\ \dot{w}' &= h(v,w') \\ \dot{v}'' &= g(v'', w') \end{aligned} \tag{4}$$

If all the Lyapunov exponents of the *w′*, *v″* subsystem are less than 0, then $v'' \rightarrow v$ as $t \rightarrow \infty$. The example of eq. (4) is referred to as cascaded synchronization.

3. AMPLITUDE INDEPENDENT SYNCHRONIZATION

In the cascaded synchronization example of eq. (4), there must be at least one nonlinearity in the response system. If the amplitude of the driving signal *v* is changed, in most cases synchronization will not occur. This would be a problem if the cascaded chaotic system were used as a communication system where the chaotic signal had to be transmitted and suffered some unknown attenuation.

To maintain synchronization when the drive signal has been attenuated (or amplified), the response system must contain scale-invariant nonlinearities, nonlinear functions $f(x)$ with the property $f(Ax) = Af(x)$. One class of nonlinear functions that have this property are piecewise linear functions that have their only breakpoint at 0. A piecewise linear function consists of two or more line segments. It is also necessary to include an amplitude-dependent nonlinear function to fold the motion back into a bounded region.

Since the nonlinear folding function was amplitude dependent, it could not be included in the response system, so a full cascaded response system could not be built. By the proper choice of nonlinear folding function, however, it was possible to build a nonlinear response system that had some of the desirable properties of a cascaded response system. The nonlinear folding function produces the signal u_d which drives the nonlinear response system. This is similar to the work of Kocarev and Parlitz [10] in which the driving signal may be some function of the variables of the chaotic drive system.

4. CIRCUIT EXAMPLE

The chaotic circuit described below fulfills all of the requirements described in the previous section. The nonlinear folding function is $g_1(y)$, while amplitude-invariant nonlinearities are provided by $g_2(x)$ and $g_3(y)$. The equations for the circuit are:

$$
\begin{aligned}
\frac{dx}{dt} &= -\alpha\left(0.05x + 0.05g_1(y) + 1.47z + 0.1S_I\right) \\
\frac{dy}{dt} &= -\alpha\left(-0.5x - 0.44g_1(y) + 0.147y\right) \\
\frac{dz}{dt} &= -\alpha\left(-0.5g_2(x) + z - 0.5w\right) \\
\frac{dw}{dt} &= -\alpha\left(-10.0g_3(g_1(y)) + 10.0w\right)
\end{aligned}
$$

$$
\begin{aligned}
g_1(y) &: \left\{\begin{array}{ll} y \le -1.6 & g_1 = -2.5y - 7.2 \\ -1.6 < y < 1.6 & g_1 = 2.0y \\ 1.6 \le y & g_1 = -2.5y + 7.2 \end{array}\right\} \\
g_2(x) &: \left\{\begin{array}{ll} x \le 0 & g_2 = 0 \\ x > 0 & g_2 = 4.5x \end{array}\right\} \qquad (5) \\
g_3(y) &: \left\{\begin{array}{ll} y \le 0 & g_3 = 4.5y \\ y > 0 & g_3 = 0 \end{array}\right\}
\end{aligned}
$$

where the time factor $\alpha=10^4\ s^{-1}$. S_I represents an information signal that may be injected into the circuit. Kocarev and Parlitz [10] also used this method to encode information on a chaotic carrier. Figure 1 is a chaotic attractor from this circuit. The largest Lyapunov exponent for this circuit was calculated numerically by the method of Eckmann and Ruelle [16] to be 765 s^{-1}.

The response circuit was driven by a scaled version of $g_1(y)$. The response circuit equations were:

$$
\begin{aligned}
&u_d = Ag_1(y)\\
&\frac{dx'}{dt} = -\alpha\left(0.05x' + 0.05u_d + 1.47z'\right)\\
&\frac{dy'}{dt} = -\alpha\left(-0.5x' - 0.44u_d + 0.147y'\right)\\
&\frac{dz'}{dt} = -\alpha\left(-0.5g_2(x') + z - 0.5w'\right)\\
&\frac{dw'}{dt} = -\alpha\left(-10.0g_3(u_d) + 10.0w'\right)
\end{aligned}
\tag{6}
$$

where A is a scaling factor that may be greater or less than 1.0. The largest Lyapunov exponent for the response circuit, calculated numerically from eq. (6), was -1470 s^{-1}, independent of the value of A.

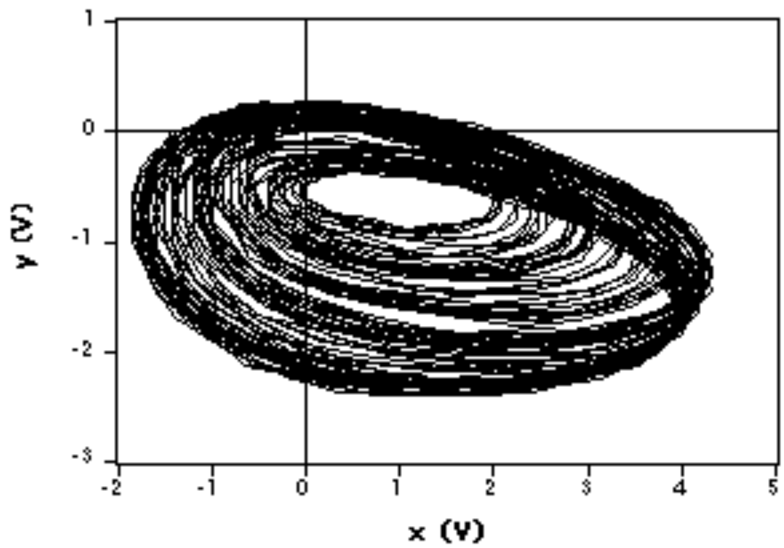

Fig. 1 Chaotic attractor from the circuit.

Synchronization of the drive and response circuits is confirmed in Fig. 2(a), which is a plot of y' vs y from the circuit, for A=1.0. When A is not 1, y' is a scaled version of y, as may be seen in Fig. 2(b), which shows y' vs y when A=0.2. Figure 3 shows the attractor for the response circuit when A=0.5. This attractor is just a scaled version of the drive system attractor of Fig. 1.

5. COMMUNICATIONS

In order to send information from the drive to the response, it is not enough merely to synchronize the y and y' signals. It must be possible to determine whether or not the systems are synchronized by comparing the drive signal u_d to the y' signal. The folding function $g_1(y)$ can be chosen so Synchronization may be detected by checking the value of u_d when y' crosses 0; if u_d =0 at this time, the systems are synchronized. This does limit the rate at which information may be transmitted to the average rate at which y' crosses 0, about 2 KHz for this system.

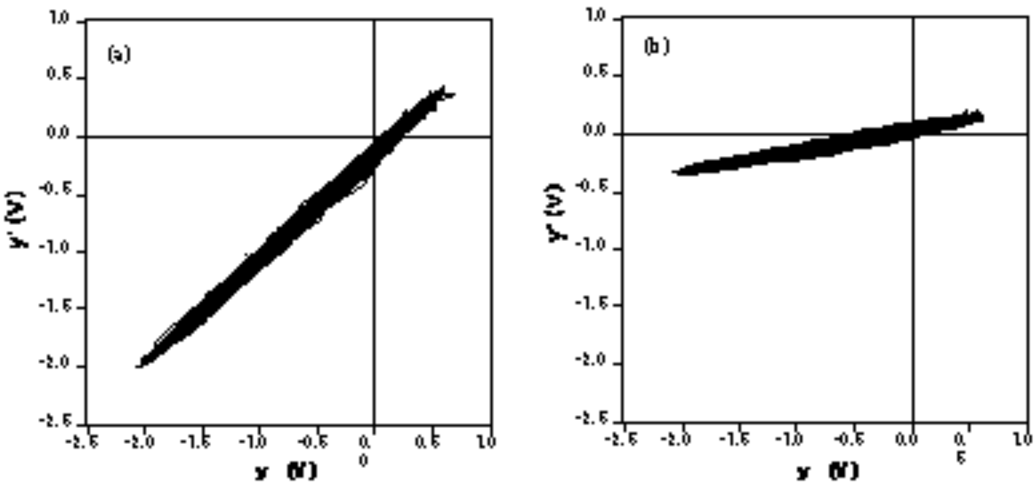

Fig. 2. (a) y' signal from the response circuit vs y signal from the drive circuit, showing synchronization, when the scaling factor A from eq. (6) is 1.0. (b) y' signal from the response circuit vs y signal from the drive circuit when A=0.2, showing that y' is a scaled down version of y .

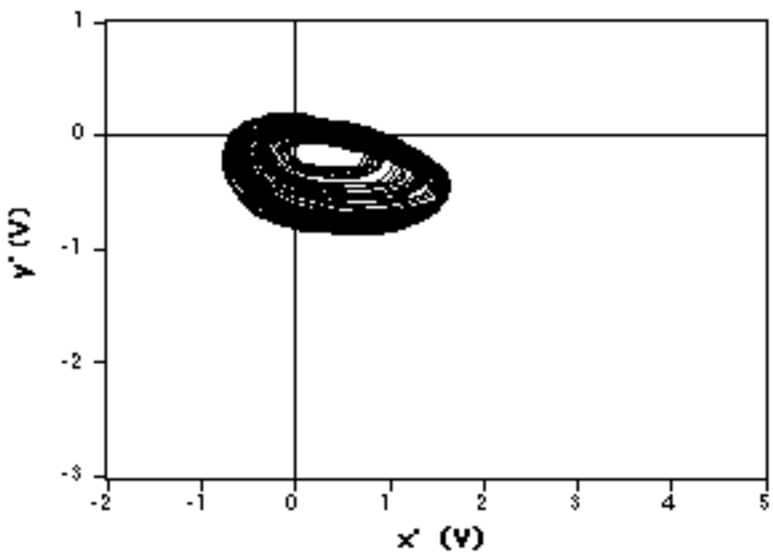

Fig. 3. Chaotic attractor from the response circuit when the scaling factor A from eq. (6) is 0.5, showing that this attractor is a scaled down version of the drive circuit attractor in Fig. 1.

There are many ways in which information may be encoded on a chaotic carrier. In this work, the information is added into the dynamical system as the signal S_I in eq. (5). This type of encoding was used because it does not have a large effect on the amplitude of the chaotic signal u_d, since the response circuit is not sensitive to changes in the amplitude of u_d. Kocarev and Parlitz [10] have noted that it is difficult to detect this kind of signal encoding in the power spectrum of the drive signal, and the same effect is seen here. The form of the information signal was $S_I = 2.0\sin(\pi f_I t)$, where f_I, the frequency of the information signal, ranged from 5 to 200 Hz. The maximum information frequency was limited by the largest negative Lyapunov exponent of the response system ($-1470\ s^{-1}$), which governed how fast the response could track a changing drive signal.

In order to detect the information in the chaotic carrier signal u_d, the negative-going zero crossings of the signal y' from the response circuit were used to strobe u_d, generating an detected signal Δ . This is essentially the same process that one would use to generate a Poincare section, but Δ ($=\Delta(t)$) was used as a 1-dimensional time series.

A low frequency power spectrum for u_d when the scaling factor A=1.0 and the information frequency f_I = 30 Hz was recorded. The signal to noise ratio at the information frequency in the power spectrum of u_d was measured by subtracting the average signal power (in dB) within 2 Hz of f_I (not including f_I) from the signal power at f_I . The signal to noise ratio was 4 dB.

A low frequency power spectrum of the detected signal Δ. when f_I = 30 Hz was also recorded. The signal to noise ratio at the information frequency was 23 dB. When the scaling factor A was 0.5, the signal to noise ratio at the information frequency was still 22 dB. For a small enough A, the signal to noise ratio will degrade due to circuit mismatch and noise.

6. NOISE AND FILTERING

In order to be useful for communications, the response circuit must not be too sensitive to noise. The two types of noise that were considered here were additive deterministic noise near the chaotic carrier frequency and low frequency additive noise.

The most difficult type of noise to separate from a chaotic carrier signal should be a signal from another chaotic system with a similar frequency spectrum. A contaminating signal ψ with a frequency spectrum similar to the spectrum of u_d may be generated by a Rossler circuit[17] described by the relations:

$$
\begin{aligned}
\frac{d\xi}{dt} &= -\alpha\left(\Gamma\xi + \beta\psi + \lambda\zeta\right)\\
\frac{d\psi}{dt} &= -\alpha\left(-\xi - \gamma\psi\right)\\
\frac{d\zeta}{dt} &= -\alpha\left(-g(\xi) + \zeta\right)\\
g(\xi) &= \begin{Bmatrix} 0 & \xi \le 3 \\ \mu(\xi - 3) & \xi > 3 \end{Bmatrix}
\end{aligned}
\tag{7}
$$

where the time factor α is $10^4\ s^{-1}$, Γ is 0.05, β is 0.5, λ is 1.0 , γ is 0.133 and μ is 15. The ψ signal was added to u_d so that the rms amplitude of ψ was up to 1.4 times the rms amplitude of u_d.

When the contaminating signal ψ was added to u_d, an information frequency f_I of 10 Hz was used. Lower information frequencies improve the signal to noise ratio in the detected signal Δ because each cycle of the information signal is avreaged over more zero crossings of y'. When f_I was 10 Hz and the rms amplitude of ψ was 1.4 times the rms amplitude of u_d, the signal to noise ratio of the detected signal Δ at the information frequency was 15 dB (with no added noise the signal to noise ratio was 26 dB). The signal to noise ratio decreased rapidly for larger amplitudes of the contaminating signal. It is possible to recover the information signal when a contaminating signal is present because the contaminating signal ψ is generated by a chaotic system that is not too similar to the drive system of eq. (5). The part of the output signal y' from the response circuit caused by ψ is not correlated with ψ, so this contribution will average to zero (unless the contaminating signal is large enough to substantially alter the dynamics of the response circuit). This effect has been demonstrated before [18, 19]

Low frequency noise has a more drastic effect on information recovery because it cannot be averaged away. In this case, it can be filtered out, since the chaotic signal u_d contains little power at low frequencies. An SR-560 preamp set to have a gain of 1 was used as a high pass filter that rolled off frequencies below 300 Hz at 12 dB/octave. The signal u_d (with an information frequency of 10 Hz) was passed through this filter before driving the response system. This filter did not have much effect on u_d except that it removed a DC bias in u_d. This could be corrected for in an adaptive fashion by adding a variable DC bias to the signal used to drive the response circuit and adjusting the bias to optimize the match between y' and this input signal (which was a filtered version of u_d). When this adaptive bias was used, no loss in signal to noise ratio for a filtered signal with an information frequency of 10 Hz was seen; the ratio after filtering vs 27 dB, compared to a ratio of 26 dB without filtering. If no bias adjustment was used, the signal to noise ratio was 19 dB. Presumably this high-pass filtering is possible because the injected information signal frequency is mixed with the other frequencies in the chaotic attractor, so that information is carried in a continuous band of intermodulation frequencies. An engineering analogy would be shifting up the frequency of an information signal by combining it in a nonlinear fashion with a higher frequency carrier. The chaotic response circuit then serves as a demodulator, recovering the information signal.

7. CONCLUSIONS

It has been shown that unknown amplitude variations in the amplitude of a chaotic driving signal need not be a problem when communicating with synchronized chaos. There are still other types of distortion that may disrupt chaotic communications signals, but the incredible variety of nonlinear systems that may be designed suggests that it may be possible to overcome other problems as well.

It has also been shown that amplitude independent chaotic synchronization may be used to transmit information, even in the presence of large amounts of noise. Not only is communication in the presence of noise necessary for practical use, it also offers some advantage in shielding signals from eavesdroppers. It has been shown that it is possible to extract messages from chaotic signals by estimating some part of the message-free chaotic signal [20, 21]. Adding chaotic noise to a chaotic carrier signal might make estimating the chaotic system dynamics more difficult, making it harder to extract the message.

8. REFERENCES

1. L. M. Pecora and T. L. Carroll, "Synchronization in Chaotic Systems," *Phys. Rev. Lett.*, Vol. 64, pp. 821-824, (1990).

2. L. M. Pecora and T. L. Carroll, "Driving Systems with Chaotic Signals," *Phys. Rev. A*, Vol. 44, pp. 2374-2383, (1991).

3. T. L. Carroll and L. M. Pecora, "Cascading Synchronized Chaotic Systems," *Physica D*, Vol. 67, pp. 126-140, (1993).

4. K. M. Cuomo, A. V. Oppenheim and S. H. Strogatz, "Synchronization of Lorenz-Based Chaotic Circuits with Applications to Communications," *IEEE Trans. CAS*, Vol. 40, pp. 626-633, (1993).

5. K. M. Cuomo and A. V. Oppenheim, "Circuit Implementation of Synchronized Chaos with Applications to Communications," *Phys. Rev. Lett.*, Vol. 71, pp. 65-68, (1993).

6. L. O. Chua, L. Kocarev, K. Eckart and M. Itoh, "Experimental Chaos Synchronization in Chua's Circuit," *Int. J. Bifurc. and Chaos*, Vol. 2, pp. 705-708, (1992).

7. H. Dedieu, M. P. Kennedy and M. Hasler, "Chaos Shift Keying: Modulation and Demodulation of a Chaotic Carrier Using self Synchronizing Chua's Circuits," *IEEE Trans. CAS*, Vol. 40, pp. 634-642, (1993).

8. R. He and P. G. Vaidya, "Analysis and Synthesis of Synchronous Periodic and Chaotic Systems," *Phys. Rev. A*, Vol. 46, pp. 7387-7392, (1992).

9. L. Kocarev, K. S. Halle, K. Eckert, L. O. Chua and U. Parlitz, "Experimental Demonstration of Secure Communications via Chaotic Synchronization," *Int. J. Bifurc. and Chaos*, Vol. 2, pp. 709-713, (1992).

10. L. Kocarev and U. Parlitz, "General Approach for Chaotic Synchronization with Applications to Communication," *Phys. Rev. Lett.*, Vol. 74, pp. 5028-5031, (1995).

11. K. Murali and M. Lakshmanan, "Transmission of signals by synchronization in a chaotic Van der Pol-Duffing oscillator," *Phys. Rev. E*, Vol. 48, pp. 1624-1626, (1993).

12. K. Pyragas, "Predictable Chaos in Slightly Perturbed Unpredictable Chaotic Systems," *Phys. Lett. A*, Vol. 181, pp. 203-210, (1993).

13. N. F. Rul'kov, A. R. Volkovski, A. Rodriguez-Lozano, E. DelRio and M. G. Velarde, "Synchronous chaotic behavior of a response oscillator with chaotic driving," *Chaos, Solitons and Fractals*, Vol. 4, pp. 201-211, (1994).

14. M. d. S. Viera, A. J. Lichtenberg and M. A. Lieberman, "Synchronization of Regular and Chaotic Systems," *Phys. Rev. A*, Vol. 46, pp. R7359-R7362, (1992).

15. T. L. Carroll, "Amplitude-independent chaotic synchronization," *submitted to Phys. Rev. E*, (1995).

16. J.-P. Eckmann and D. Ruelle, "Ergodic Theory of Chaos and Strange Attractors," *Rev. Mod. Phys.*, Vol. 57, pp. 617-656, (1985).

17. T. L. Carroll, "A simple circuit for demonstrating regular and synchronized chaos," *Am. J. Phys.*, Vol. 63, pp. 377-379, (1995).

18. T. L. Carroll, "Synchronizing Chaotic Systems Using Filtered Signals," *Phys Rev. E*, Vol. 50, pp. 2580-2587, (1994).

19. T. L. Carroll, "Communicating using Filtered Synchronized Chaotic Signals," *IEEE Trans. CAS*, Vol. 42, pp. 105-110, (1995).

20. K. M. Short, "Steps toward unmasking secure communications," *Int. J. Bifurc. and Chaos*, Vol. 4, pp. 959-977, (1994).

21. G. Perez and H. A. Cerdeira, "Extracting Messages Masked by Chaos," *Phys. Rev. Lett.*, Vol. 74, pp. 1970-1973, (1995).